Advanced topics related to the volume integral equation formulation of electromagnetic scattering

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The frequency-domain volume integral equation (VIE) for the electric field inside the scattering object has been known for more than 50 years. It has been intended to be a rigorous formulation of the electromagnetic-scattering problem equivalent to the more conventional one based on the differential Maxwell equations subject to appropriate boundary conditions. Moreover, the VIE has been used as the fundamental basis for a number of “numerically exact” computational methods to simulate electromagnetic scattering, the most popular one being the discrete dipole approximation. The latter has been successfully used for virtually all classes of scatterers, including those with sharp edges and internal interfaces.

Despite the vast existing literature on the subject, the theoretical understanding of the VIE remains incomplete and incommensurate to the domain of its actual practical applications. Indeed, the literature is largely grouped around the following two extremes: (i) accessible derivations with all complex issues swept under the rug with the intent to maximally shorten the path to practical computations, and (ii) mathematically rigorous yet practically limited treatises that commence with concepts such as Banach spaces, Hölder continuity, etc. and thus are hardly comprehensible to the applied scattering community. As a consequence, the publications from the first group tend to ignore fundamental issues such as the strong singularity of the integral kernel, which can potentially lead to ambiguities. And rigorous mathematical studies are typically based on simplified assumptions of smooth particle boundaries and continuous interiors, whereas sharp edges/vertices and internal interfaces are hardly mentioned. In addition, the current understanding of the conditions guarantying the existence and uniqueness of the VIE solution remains fragmentary, especially in the case of an absorbing host medium.

To fill these essential gaps, we present an accessible and general derivation of the VIE from the differential Maxwell equations, transmission boundary conditions, and locally-finite-energy condition with an explicit treatment of the kernel singularity. Our derivation applies to a representative type of scattering object such as a spatially finite group of multi-layered particles with piecewise smooth (intersecting) boundaries and internal interfaces (with a smooth refractive index in between) immersed in a passive unbounded host medium. We also generalize the results of existing mathematical analyses of the VIE and formulate a conjecture about sufficient conditions ensuring the existence and uniqueness of its solution for this type of scatterer. Finally, we discuss an alternative way of deriving the VIE for an arbitrary object with discontinuities by means of a continuous transformation of the everywhere smooth refractive-index function into a discontinuous one.

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